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EVENT-TRIGGERING OF LARGE-SCALE SYSTEMS WITHOUT ZENO BEHAVIOR

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Abstract. We present a Lyapunov based approach to event-triggering for large-scale systems using a small gain argument. The problem of Zeno behavior is resolved by requiring practical stability of the closed loop system.

Key words. Networked Control Systems, Event-Triggering, Small-Gain Theorem, Nonlinear Systems

1. Introduction. As control systems become increasingly complex, the aspect of communication in control systems is more and more relevant. In this paper we continue our work from [4] and consider the problem of decentralized event-triggered control, which is prominent in networked control systems, [1, 9, 10, 12]. We consider large-scale interconnections of controlled systems and present a method for reducing the necessary amount of communication. It is assumed that information exchange between measurement devices and controllers takes place over a limited communication channel. To reduce the information load we introduce a small gain approach to event-triggering which leads to practical stability of the system and ensures that Zeno behavior does not occur. In particular, the triggering condition is constructed in a systematic manner using local Lyapunov functions.
One drawback of event-triggering is the need for continuously monitoring the state. An approach that tries to overcome this issue is termed self-triggering [8, 11]. In hybrid systems, the practice of avoiding Zeno effects while retaining stability in the practical sense is referred to as temporal regularization (see [7], p. 73, and references therein). Here, the regularization is achieved via a notion of practical ISS. In the context of event-triggered $\mathcal{L}_2$-disturbance attenuation control for linear systems temporal regularization is studied in [6].

2. Preliminaries. Notation Let $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. The set of nonnegative reals is denoted by $\mathbb{R}_+$, and $\mathbb{R}^n_+$ is the nonnegative orthant, i.e. $\mathbb{R}^n_+ := \{x \in \mathbb{R}^n : x_i \geq 0, 1 \leq i \leq n\}$. By $||\cdot||$ we denote the Euclidean norm of a vector.
A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ is a class-$\mathcal{K}$ function if it is continuous, strictly increasing and $\alpha(0) = 0$. If, in addition, it is unbounded, then α is said to be of class-$\mathcal{K}_\infty$. In the notation $\mathcal{K} \cup \{0\}$ ($\mathcal{K}_\infty \cup \{0\}$) the 0 refers to the function which is identically zero. A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ is positive definite if $\alpha(r) = 0$ if and only if $r = 0$.
In this paper we investigate event-triggered control schemes. Such schemes (or similar ones) have been studied in [1, 8, 9, 10, 11, 12].
Consider the interconnection of N systems described by equations of the form:

$$\dot{x}_i = f_i(x, u_i)\,, \quad u_i = g_i(x + e)\,, \tag{2.1}$$

where $i \in \mathcal{N} := \{1, 2, \ldots, N\}$, $x = (x_1^\top \ldots x_N^\top)^\top$, with $x_i \in \mathbb{R}^{n_i}$, is the state and $u_i \in \mathbb{R}^{m_i}$ is the ith control input. The systems dynamics is given by f_i and g_i is a

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specified controller. Also $e = (e_1^\top \dots e_N^\top)^\top$ and $e_i \in \mathbb{R}^{n_i}$, represents an error, which results from a possible imperfect knowledge of the controller of the state of the system. We assume that the maps f_i satisfy appropriate conditions guaranteeing existence and uniqueness of solutions for $\mathcal{L}_\infty$ inputs e. It is convenient to define sets, which describe which states are used by which controller: $C(i) = \{j \in \mathcal{N} \ : \ g_i \text{ depends explicitly on } x_j\}$ and $i \in Z(j) :\Leftrightarrow j \in C(i)$. System (2.1) combined with a triggering scheme (T_i) has the form

$$\begin{aligned} &\dot{x}_i = f_i(x, u_i)\,, \quad u_i = g_i(x+e) \\ &\dot{\hat{x}} = 0\,, \qquad\quad e = \hat{x} - x\,, \qquad T_i(x_i, e_i) \geq 0\,. \end{aligned} \tag{2.2}$$

Here x_i is the state of system $i \in \mathcal{N}$, $\hat{x}$ is the information available at the controller and the controller error is $e = \hat{x} - x$. We assume that the triggering function T_i are jointly continuous in x_i, e_i and satisfy $T_i(x_i, 0) < 0$ for all $x_i \neq 0$.
Solutions to such a triggered feedback are defined as follows. We assume that the initial controller error is $e_0 = 0$. Given an initial condition x_0 we define

$$t_1 := \inf\{t > 0 \ : \ \exists i \in \mathcal{N} \text{ s.t. } T_i(x_i(t), e_i(t)) \geq 0\}\,.$$

At time instant t_1 the systems i for which $T_i(x_i, e_i) \geq 0$ broadcast their respective state x_i to all controllers g_j with $i \in C(j)$. The systems j which use the state x_i in the control law $g_j(x)$ update only the state x_i while all the other variables are kept equal to the previously set values. Then inductively we set for $k = 1, 2, \dots$

$$t_{k+1} := \inf\{t > t_k \ : \ \exists i \in \mathcal{N} \text{ s.t. } T_i(x_i(t), e_i(t)) \geq 0\}\,.$$

We say that the triggering scheme induces Zeno behavior if for a given initial condition x_0 the event times t_k converge to a finite t^*.

3. Small-Gain Results for Large-Scale Systems. Before stating the stability assumption on each subsystem, we have to define in which way the influence of different systems on a single subsystem are compared.

Definition 3.1. *A continuous function $\mu : \mathbb{R}_+^n \to \mathbb{R}_+$ is a* monotone aggregation function *if:*
(i) $\mu(v) \geq 0$ for all $v \in \mathbb{R}_+^n$ and $\mu(v) = 0$ iff $v = 0$;
(ii) $\mu(v) > \mu(z)$ if $v > z$ and $\mu(v) \to \infty$ as $||v|| \to \infty$.
The space of monotone aggregate functions (MAFs in short) with domain $\mathbb{R}_+^n$ is denoted by MAF_n. Moreover, we say $\mu \in MAF_n^m$ if $\mu_i \in MAF_n$ for $i = 1, \dots, m$. The stability assumption for each subsystem is now given by

Assumption 1. *For $i \in \mathcal{N}$, there exist a differentiable function $V_i : \mathbb{R}^{n_i} \to \mathbb{R}_+$, and class-$\mathcal{K}_\infty$ functions α_{i1}, α_{i2} such that*

$$\alpha_{i1}(||x_i||) \leq V_i(x_i) \leq \alpha_{i2}(||x_i||)\,.$$

Moreover there exist functions $\mu_i \in \mathrm{MAF}_{2N}$, $\gamma_{ij}, \eta_{ij} \in \mathcal{K}_\infty$, for $j \in \mathcal{N}$, positive definite functions α_i and positive constants c_i, for $i \in \mathcal{N}$, such that

$$\begin{aligned} V_i(x_i) \geq \max\{\mu_i\big(\gamma_{i1}(V_1(x_1)), \dots, \gamma_{iN}(V_N(x_N))\big), \eta_{i1}(||e_1||), \dots, \eta_{iN}(||e_N||)), c_i\} \\ \Rightarrow \nabla V_i(x_i) f_i(x, g_i(x+e)) \leq -\alpha_i(||x_i||)\,. \end{aligned} \tag{3.1}$$

The comparison functions γ_{ij} appearing in (3.1) are called *gains* whereas the constants c_i are *offsets*. Assumption 1 amounts to saying that each subsystem is input-to-state

practically stable with respect to the influence of other subsystems and the error e. This describes the effect of the imperfect knowledge of the controllers.
Of course, it is not sufficient that each single subsystem is stable to infer stability of a interconnected system. Hence we have to give further conditions, which will ensure the desired stability property of the overall system. To this end define

$$\Gamma := \begin{pmatrix} 0 & \gamma_{12} & \cdots & \gamma_{1N} \\ \vdots & \ddots & \ddots & \vdots \\ \gamma_{N1} & \cdots & \gamma_{NN-1} & 0 \end{pmatrix}$$

as the matrix with the gains from Assumption 1, which describes the topology of the interconnection in the sense that a subsystem i influences subsystem j directly if and only if $\gamma_{ji} \neq 0$. We can associate in an obvious manner a graph to this interconnection topology. We will say that Γ is irreducible if the associated graph is strongly connected. Such a gain matrix acts on $s \in \mathbb{R}^n_+$ through $\mu \in \mathrm{MAF}^N_N$ by

$$\Gamma_\mu(s) := \begin{pmatrix} \mu_1(0, \gamma_{12}(s_2), \ldots, \gamma_{1N}(s_N)) \\ \vdots \\ \mu_N(\gamma_{N1}(s_1), \ldots, \gamma_{NN-1}(s_{N-1}), 0) \end{pmatrix} .$$

The stability of an interconnected system can now be inferred by a small-gain argument. In this work the notion of small-gain is contained in the following definition.

DEFINITION 3.2. *A map $\sigma : \mathbb{R}_+ \to \mathbb{R}^N_+$, $\sigma \in \mathcal{K}^N_\infty$, is an Ω-path with respect to Γ_μ if:*

(i) for each i, the function σ_i^{-1} is locally Lipschitz continuous on $(0,\infty)$;

(ii) for every compact set $K \subset (0,\infty)$ there are constants $0 < c < C$ such that for all $i = 1, 2, \ldots, N$ and all points of differentiability of σ_i^{-1} we have:

$$0 < c \leq (\sigma_i^{-1})'(r) \leq C\ , \quad \forall r \in K;$$

(iii) $\Gamma_\mu(\sigma(r)) < \sigma(r)$ for all $r > 0$.

The relation of existence of an Ω-path to small-gain results is given by:

THEOREM 3.3. *Let $\Gamma \in (\mathcal{K}_\infty \cup \{0\})^{N\times N}$ and $\mu \in \mathrm{MAF}^N_N$. If Γ is irreducible and $\Gamma_\mu \not\geq id$ [1] then there exists an Ω-path σ with respect to Γ_μ.* For a discussion and a proof for Theorem 3.3 see [3]. To account for the imperfect knowledge of the state to the controller, we have to add a condition which accounts to the existence of the error e.

ASSUMPTION 2. *Let μ be as in Assumption 1 and $\overline{\mu}(a) := \mu(a, 0)$ for all $a \in \mathbb{R}^N_+$. There exists an Ω-path σ with respect to $\Gamma_{\overline{\mu}}$. Furthermore, there exists a map $\varphi \in (\mathcal{K}_\infty \cup \{0\})^{N\times N}$ such that $\varphi_{ij} \neq 0$ if and only if $j \in C(i)$ and*

$$\overline{\Gamma}_\mu(\sigma(r), \varphi(r)) < \sigma(r)\ , \quad \forall r > 0\ , \tag{3.2}$$

where $\overline{\Gamma}_\mu(\sigma(r), \varphi(r))$ is defined by

$$\overline{\Gamma}_\mu(\sigma(r), \varphi(r)) := \begin{pmatrix} \mu_1(\gamma_{11}(\sigma_1(r)), .., \gamma_{1n}(\sigma_N(r)), \varphi_{11}(r), .., \varphi_{1N}(r)) \\ \vdots \\ \mu_N(\gamma_{N1}(\sigma_1(r)), .., \gamma_{NN}(\sigma_N(r)), \varphi_{N1}(r), .., \varphi_{NN}(r)) \end{pmatrix}$$

REMARK 1. *The existence of the mapping φ is not restrictive. This can easily be seen, if $\mu = \max$. For the case of a general* MAF *see [5].*

[1] $\Gamma_\mu \not\geq id$ means that for all $s \in \mathbb{R}^N_+$, $s \neq 0$ we have $\Gamma_\mu(s) \not\geq s$, i.e. $\exists i \in \mathcal{N}$: $\mu_i(s_1, \ldots, s_N) < s_i$.

4. Main Result. In this section we introduce the approach to event triggering. We first show that a certain condition on the error e together with a small gain assumption ensures that trajectories are decreasing along a certain Lyapunov function. It is then shown that this assumption can be guaranteed by following a certain triggering scheme, which results in the design of a practically stabilizing event-triggering scheme. As the Lyapunov function V under consideration is only Lipschitz continuous we consider the Clarke gradient $\partial V(x)$ at x.

THEOREM 4.1. *Let Assumptions 1 and 2 hold. Let $V(x) = \max_{i\in\mathbb{N}} \sigma_i^{-1}(V_i(x_i))$. Assume that for each $j \in \mathbb{N}$,*

$$\max\{\sigma_j^{-1}(V_j(x_j)), c_j\} \geq \hat{\eta}_j(\|e_j\|) \ , \quad \textit{with } \hat{\eta}_j = \max_{i\in Z(j)} \varphi_{ij}^{-1} \circ \eta_{ij} \ . \tag{4.1}$$

Then there exists a positive definite $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$\langle p, f(x, g(x+e))\rangle \leq -\alpha(||x||), \ \forall p \in \partial V(x) \ ,$$

for all $x = (x_1^\top \, x_2^\top \, \dots \, x_N^\top)^\top \ \in \ \{x : V(x) \geq \hat{c} := \max_i\{c_i, \sigma_i^{-1}(c_i)\}\}$, where

$$f(x, g(x+e)) = \begin{pmatrix} f_1(x, g_1(x+e)) \\ \dots \\ f_N(x, g_N(x+e)) \end{pmatrix} .$$

Proof. Let $\mathcal{N}(x) \subseteq \mathcal{N}$ be the indices i such that $V(x) = \sigma_i^{-1}(V_i(x_i))$.
Take any pair of indices $i, j \in \mathcal{N}$. By definition, $V(x) \geq \sigma_j^{-1}(V_j(x_j))$ and

$$\gamma_{ij}(\sigma_j(V(x))) \geq \gamma_{ij}(V_j(x_j)) \ . \tag{4.2}$$

Let $i \in \mathcal{N}(x)$. Then by Assumption 2, we have:

$$\begin{aligned} V_i(x_i) = \sigma_i(V(x)) > \\ \mu_i(\gamma_{i1}(\sigma_1(V(x))), \dots, \gamma_{iN}(\sigma_N(V(x))), \varphi_{i1}(V(x)), \dots, \varphi_{iN}(V(x))) \ . \end{aligned} \tag{4.3}$$

Bearing in mind (4.2), we also have

$$\begin{aligned} V_i(x_i) = \sigma_i(V(x)) > \\ \mu_i(\gamma_{i1}(V_1(x_1)), \dots, \gamma_{iN}(V_N(x_N)), \varphi_{i1}(V(x)), \dots, \varphi_{iN}(V(x))) \ . \end{aligned} \tag{4.4}$$

Let us partition the set $\mathcal{N} := \mathcal{P} \cup \mathcal{Q}$. The set $\mathcal{P}$ consists of all the indices i for which the first part of the maximum in condition (4.1) holds, i.e. $i \in \mathcal{P} :\Leftrightarrow \sigma_i^{-1}(V_i(x_i)) \geq c_i$; also $\mathcal{Q} := \mathcal{N} \setminus \mathcal{P}$. For all $j \in \mathcal{P}$ we have by (4.1) $\sigma_j^{-1}(V_j(x_j)) \geq \hat{\eta}_j(\|e_j\|)$ and hence using (4.1) (the case $j \notin C(i)$ is trivial because in this case $\varphi = \eta = 0$)

$$\begin{aligned} \varphi_{ij}(V(x)) \geq \varphi_{ij} \circ \sigma_j^{-1}(V_j(x_j)) \geq \varphi_{ij} \circ \hat{\eta}_j(\|e_j\|) \geq \\ \varphi_{ij} \circ \varphi_{ij}^{-1} \circ \eta_{ij}(\|e_j\|) = \eta_{ij}(\|e_j\|) \ . \end{aligned} \tag{4.5}$$

Assume now that $V(x) \geq \hat{c}$. For all $j \in \mathcal{Q}$ we have by (4.1) $c_j \geq \hat{\eta}_j(\|e_j\|)$ and so

$$\varphi_{ij}(V(x)) \geq \varphi_{ij}(\hat{c}) \geq \varphi_{ij}(c_j) \geq \varphi_{ij} \circ \hat{\eta}_j(\|e_j\|) \geq \eta_{ij}(\|e_j\|). \tag{4.6}$$

Combining (4.5) and (4.6) we get for all $j \in \mathcal{N}$ $\varphi_{ij}(V(x)) \geq \eta_{ij}(\|e_j\|)$, provided that (4.1) holds and that $V(x) \geq \hat{c}$. Substituting the latter in (4.4) yields

$$\begin{aligned} V_i(x_i) = \sigma_i(V(x)) > \\ \mu_i(\gamma_{i1}(V_1(x_1)), \dots, \gamma_{iN}(V_N(x_N)), \eta_{i1}(||e_1||), \dots, \eta_{iN}(||e_N||)) \ . \end{aligned} \tag{4.7}$$

Because $i \in \mathcal{N}(x)$ and $V(x) \geq \hat{c} = \max_i\{c_i, \sigma_i^{-1}(c_i)\}$, we have $V(x) = \sigma_i^{-1}(V_i(x_i)) \geq \hat{c} \geq \sigma_i^{-1}(c_i)$ and finally we conclude $V_i(x_i) \geq c_i$. The latter together with (4.7) is the left-hand side of the implication (3.1). Hence, for all $i \in \mathcal{N}(x)$: $\nabla V_i(x_i) f_i(x, g_i(x+e)) \leq -\alpha_i(||x_i||)$. Now we can repeat the same arguments of the last part of the proof of Theorem 5.3 in [3], and conclude that for all x such that $V(x) \geq \hat{c}$ and for all $p \in \partial V(x)$, $\langle p, f(x, g(x+e))\rangle \leq -\alpha(||x||)$. ◻ Now that we established the existence of a Lyapunov function for the overall system, we show that with a suitable triggering scheme the conditions of Theorem 4.1 holds. We stress that the information needed for the triggering condition is purely local.

THEOREM 4.2. *Let Assumptions 1 and 2 hold. Consider the interconnected system*

$$\dot{x}_i(t) = f_i(x(t), g_i(\hat{x}(t))) \ , \quad i \in \mathbb{N} \ , \tag{4.8}$$

as in (2.2) *with triggering conditions given by*

$$T_i(x_i, e_i) = \hat{\eta}_i(\|e_i\|) - \max\{\sigma_i^{-1} \circ V_i(x_i), \hat{c}_i\} \ , \tag{4.9}$$

with $\hat{\eta}_i$ defined in (4.1) *for all $i \in \mathcal{N}$. Then the origin is a globally uniformly practically stable equilibrium for (4.8). In particular, no Zeno behavior is induced.*

Proof. The triggering condition (4.9) ensures that (4.1) holds. Hence there exists a Lyapunov function for the interconnected system. By standard arguments stability of the system follows. (see e.g. [2]). It remains to show, that no Zeno behavior is induced. In between triggering events $\dot{e}(t) = -\dot{x}(t)$ for all $t \in (t_k, t_{k+1})$ by (2.2). $V(x(t))$ is decreasing along the solution $x(t)$ on its domain of definition. Hence, $x(t)$ is bounded on its domain of definition. Since $\max\{\sigma_i^{-1} \circ V_i(x_i(t)), \hat{c}_i(t)\} \geq \hat{\eta}_i(\|e_i(t)\|)$, then also $e(t)$ is bounded and so is $\hat{x}(t) = x(t) + e(t)$. As $e_j(t_k^+) = 0$ for each index j which triggered an event and $\dot{e}(t)$ is bounded in between events ($\dot{e}(t) = -\dot{x}(t) = -f(x(t), g(\hat{x}(t)))$), the time when the next event will be triggered by system j is bounded away from zero because the time it takes e_j to evolve from zero to c_j is bounded away from zero. Hence, either there is a finite number of times t_k or $t_k \to \infty$ as k goes to infinity. And so no Zeno behavior can occur. ◻

5. Numerical Example. Consider the interconnection of $N = 2$ subsystems and control laws given by

$$\begin{aligned} \dot{x}_1 &= x_1 x_2 + x_1^2 u_1 & \dot{x}_2 &= x_1^2 + u_2 \ , \\ u_1 &= -k_1(x_1 + e_1) & u_2 &= -k_2(x_2 + e_2) \ , \ k_1, k_2 > 0 \ , \end{aligned}$$

and $V_i(x_i) = \frac{1}{2}x_i^2$ for $i = 1, 2$. By straightforward calculation (see [5] for details)

$$\mu_2 = \max \ , \ \gamma_{21}(r) = \frac{32}{k_2^2} r^2 \ , \ \gamma_{22}(r) = 0 \ , \eta_{21}(r) = 0 \ , \ \eta_{22}(r) = 8r^2 \ . \tag{5.1}$$

The Ω-path can be chosen as $\sigma_1 = Id$ and $\sigma_2(r) = \overline{\sigma}^2 r^2$ with $\overline{\sigma} \in (\frac{32}{k_2^2}, \frac{k_1^2}{32})$. Setting

$$\varphi_{11}(r) = \frac{\sqrt{32}}{k_1}\overline{\sigma} r \ , \ \varphi_{12} \equiv \varphi_{21} \equiv 0 \ , \ \varphi_{22}(r) = \frac{32}{k_2^2} r^2$$

yields $\hat{\eta}(r) := \frac{2}{\overline{\sigma}\sqrt{2}} r^2$ and $\hat{\eta}_2 := \frac{33}{2}|r|$ for $k_1 = 1$, $k_2 = 33$ and Ass. 1 and 2 hold. The offsets are chosen to be $c_1 = 35$ and $c_2 = 1.86$. Figure 5.1 depicts the Lyapunov

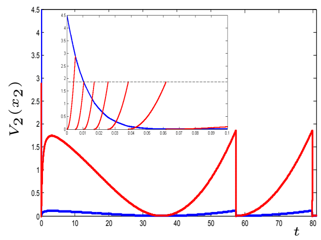

FIG. 5.1. *Lyapunov function of the subsystem (blue) together with the error (red) and zoom into* $t \in [0, 0.1]$.

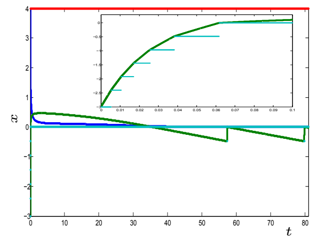

FIG. 5.2. *Zeno Trajectories of the interconnected system. Zoom into* $t \in [0, 0.1]$

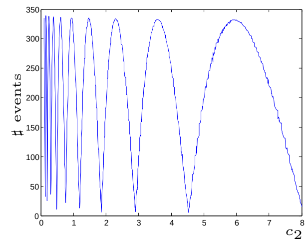

FIG. 5.3. *Offset* c_2 *against the resulting number of events for* $t \in [0, 20]$

function of the second subsystem. Whenever the red curve is bigger than the Lyapunov function (blue) and bigger as the offset c_2 from Assumption 1, an event is triggered. Because the system converges fast, a zoom into the beginning is also depicted in Figure 5.1. Figure 5.2 shows the trajectories of the system. The first system is given in blue and the second in green. The control law is calculated using the red and turquoise values accordingly. After $t \approx 40$ the second subsystem enters a stable limit cycle, with size depending on the offset c_2.

In Figure 5.3 a plot of the offset c_2 against the number of triggered events for a simulation up to $t = 20$ can be found. Interestingly, it seems that there is no easy heuristic how to choose the parameter to minimize the number of events. Note that $c_2 = 1.86$ yields the smallest number of events (10). A choice of $c_2 = 0.16$ leads to a number of 339 events on the same time horizon. Choosing the offset too small ($c_2 < 10^{-10}$) would lead to a continuum of samples around $t \approx 0.093$.

6. Conclusion. In this paper we have introduced a small-gain approach to event triggering. The triggering condition is derived using Lyapunov techniques. A condition ensuring practical stability and the nonexistence of Zeno behavior is natural in the context considered here. A refined version of this approach is under investigation.

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